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## RAPID COMMUNICATIONS

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# Irrelevance of memory in the minority game

Andrea Cavagna*

*Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, United Kingdom*

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By means of extensive numerical simulations, we show that all the distinctive features of the minority game introduced by Challet and Zhang [Physica A **256**, 514 (1998)] are completely independent of the memory of the agents. The only crucial requirement is that all the individuals must possess the same information, irrespective of whether this information is true or false. [S1063-651X(99)50204-3]

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Originally inspired by the ''El Farol'' problem stated by Arthur in [1], a model system has been introduced in [2] for the adaptive evolution of a population of interacting agents, the so-called minority game. This is a toy model where inductive, rather than deductive, thinking, in a population of bounded rationality, gives rise to cooperative phenomena.

The setup of the minority game is the following: $N$ agents have to choose at each time step whether to go in room 0 or 1. Those agents who have chosen the less crowded room (minority room) win, the others lose, so that the system is intrinsically frustrated.

A crucial feature of the model is the way by which agents choose. In order to decide in what room to go, agents use strategies. A strategy is a choosing device; that is, an object that processes the outcomes of the winning room in the last $m$ time steps (each outcome being 0 or 1) and accordingly to this information prescribes in what room to go in the next step. The so-called memory $m$ defines $2^m$ potential past histories (for instance, with $m=2$ there are four possible pasts, 11, 10, 01, and 00). A strategy is thus formally a vector $R_\mu$, with $\mu=1,\dots,2^m$, whose elements can be 0 or 1. The space $\Gamma$ of the strategies is a hypercube of dimension $D=2^m$ and the total number of strategies is $2^D$.

At the beginning of the game each agent draws randomly a number $s$ of strategies from the space $\Gamma$ and keeps them forever, as a genetic heritage. The problem is now to fix which one, among these $s$ strategies, the agent is going to use. (We will consider only the nontrivial case $s>1$.) The rule is the following. During the game the agent gives points to *all* his/her strategies according to their potential success: at each time step a strategy gets a point only if it has forecast the correct winning room, regardless of whether or not it has actually been used. At a given time the agent chooses among his/her $s$ strategies the most successful one up to that moment (i.e., the one with the highest number of points) and uses it in order to choose the room. The adaptive nature of the game relies on the time evolution of the best strategy of each single agent. In this way the game has a well-defined deterministic time evolution, which only depends on the initial distribution of strategies and on the random initial string of $m$ bits necessary to start the game.

Among all the possible observables, a special role is played by the variance $\sigma$ of the attendance $A$ in a given room [2]. We can consider, for instance, room 0 and define $A(t)$ as the number of agents in this room at time $t$. We have

$$\sigma^2=\lim_{t\to\infty}\frac{1}{t}\int_{t_0}^{t}dt'\left(A(t')-\frac{N}{2}\right)^2, \tag{1}$$

where $N/2$ is the average attendance in the room and $t_0$ is a transient time after which the process is stationary [2,3]. In all the simulations presented in this Rapid Communication

*Electronic address: a.cavagna1@physics.ox.ac.uk

 

$t = t_0 = 10\,000$ has been taken for a maximum value of $N = 101$ and it has been verified that the averages were saturated over these times.

The importance of $\sigma$ (called *volatility* in a financial context) is simple to understand: the larger $\sigma$ is, the larger the global waste of resources by the community of agents. Indeed, only with an attendance $A$ as near as possible to its average value is there the maximum distribution of points to the whole population. Moreover, from a financial point of view, it is clear that a low volatility $\sigma$ is of great importance in order to minimize the risk.

If all the agents were choosing randomly, the variance would simply be $\sigma_r^2 = N/4$. An important issue is therefore: under what conditions is the variance $\sigma$ *smaller* than $\sigma_r$? In other words, is it possible for a population of selfish individuals to collectively behave in a better-than-random way? What has been found first in [3] is that the volatility $\sigma$ as a function of $m$ has a remarkable behavior, since actually *there is* a regime where $\sigma$ is smaller than the random value $\sigma_r$. In this phase the collective behavior is such that less resources are globally wasted by the population of agents. A deep understanding of this feature is therefore important.

From the very definition of the model and from the behavior of $\sigma(m)$ described above, it seems clear that the memory $m$ is a crucial quantity for the two following reasons. First, from a geometrical point of view, $m$ defines the dimension of the space of strategies $\Gamma$ and therefore it is related to the probability that strategies drawn randomly by different agents could give similar predictions: the larger $m$ is, the bigger $\Gamma$ is and the lower the probability is that different players have some strategies in common. Since the nonrandom nature of the game relies on the presence of correlated choices, that is, exactly on the possibility that different agents use the same strategies, it follows that for very large $m$ the game proceeds in a random way [3–6]. (This argument works at a fixed number of agents $N$. Otherwise, the relevant variable will be $2^m/N$. We discuss this point later.)

Second, $m$ is supposed to be a real memory. Actually, the whole game is constructed around the role of $m$ as a memory: at time $t$ agents use strategies which process the last $m$ events in the past. As a consequence of this, a new minority room will come out and at time $t+1$ there will be a new $m$-bits past which will differ from the old one for the outcome at time $t$. Thus, agents, or better, strategies, choose by remembering the last $m$ steps of time history, so that $m$ is a natural time scale of the system. Due to this, an explanation of the behavior of $\sigma(m)$ has been proposed in [3], where the decay rate of the time correlations in the system is compared and related to $m$, thus supporting the key interpretation of $m$ as a real memory. This memory role of $m$ complicates greatly the nature of the problem, since it induces an explicits dynamical feedback in the evolution of the system, such that the process is not local in time.

The purpose of this Rapid Communication is to show that the memory of the agents is irrelevant. We shall prove that there is no need for an explicit time feedback in order to obtain all the distinctive features of the model. In order to prove this statement we consider the same model introduced in [2] and described above, but with the following important difference: at each time step, the past history is just *invented*; that is, a random sequence of $m$ bits is drawn, to play the role of a fake time history. This is the information that all the agents process with their best strategies to choose the room. As we are going to show, this oblivious version of the model gives exactly the same results as the original one, thus proving that the role of $m$ is purely geometrical.

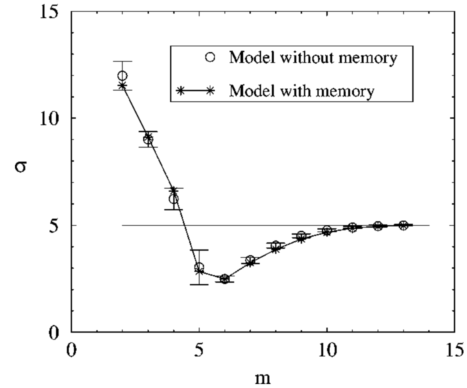


FIG. 1. Model without memory vs model with memory. The variance $\sigma$ as a function of $m$ for $s=2$. The horizontal line is the variance $\sigma_r$ of the random case. The number of agents is $N=101$. Average is over 100 samples. Errors bars are shown only for the model without memory, while the line just connects the points of the memory model.

In Fig. 1, the variance $\sigma$ as a function of $m$ is plotted for both the case with and the case without memory. The two models give the same results, not only qualitatively, but also quantitatively (see also the data of [3–6]). In particular, the minimum of $\sigma$ as a function of $m$ is found even without memory and cannot therefore be related to it.

The dependence of the whole function $\sigma(m)$ on the individual number of strategies $s$ is another important point. It has been shown for the first time in [4] the larger the value of $s$ is, the shallower the minimum of this curve is. In Fig. 2 we show that this same phenomenon occurs for the model without memory.

From a technical point of view, note that once the role of $m$ as a memory is eliminated, the only quantity involved in the actual implementation of the model is $D$, the dimension of the space of strategies $\Gamma$. Therefore, instead of drawing a

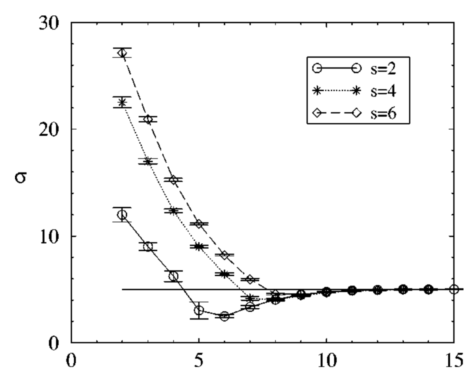


FIG. 2. Model without memory. Variance $\sigma$ as a function of $m$, at different values of $s$, $N=101$. Average is over 100 samples. Lines are just a guide for the eye.

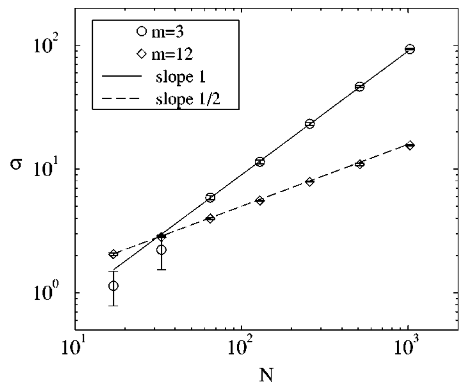


FIG. 3. Model without memory. Variance $\sigma$ as a function of the number of agents $N$, for different values of $m$, at fixed $s=2$. Average is over ten samples. The full line is $\sigma \sim N$, while the dashed line is $\sigma \sim N^{1/2}$.

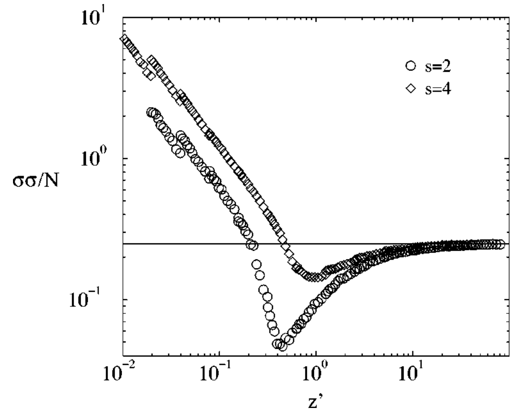


FIG. 4. Model without memory. Plot of $\sigma^2/N$ as a function of the scaling parameter $z'=2D/sN$. The number of agents $N$ varies from $N=51$ to $N=101$, while $D$ varies from $D=2$ to $D=4096$. The individual number of strategies $s$ ranges over two values $s=2$ and $s=4$. Average is over 50 samples.

random sequence of $m$ bits, it is much easier to draw a random component $\mu \in [1,D]$ to mimic the past history: each agent uses component $\mu$ of his/her best strategy to choose the room. The main consequence of this is that there is no need for being $D=2^m$, since we can choose any integer value of $D$. In [5] a method has been introduced by which it is possible to consider noninteger values of $m$ in the model with memory. This is useful, since it permits one to study the shape of $\sigma(m)$ around its minimum, with a better resolution in $m$. In the present context, it is trivial to consider noninteger values of $m$, since we simply have $m=\log_2 D$. In this way results identical to [5] are obtained.

Once $s$ is fixed, let $m_c$ be the value of $m$ where the minimum of $\sigma(m)$ occurs. In [3] it has been pointed out that for $m<m_c$ the variance $\sigma$ grows as $N$, where $N$ is the number of agents, while for $m>m_c$ it grows as $N^{1/2}$. In Fig. 3, $\sigma$ as a function of $N$ is plotted for the model without memory. The same behavior as in the model with memory is found.

An interesting question is whether $\sigma$ is a function of a single scaling variable $z$ constructed with $m$, $N$, and $s$. It has been shown in [3] that by considering as a scaling variable $z=2^m/N=D/N$ all the data for $\sigma$ at various $m$ and $N$ collapse on the same curve. In this case the relevant parameter is thus the dimension $D$ of $\Gamma$, over the number $N$ of playing strategies. On the other hand, a different scaling variable has been proposed in [4]; that is, $z'=2\times 2^m/sN=2D/sN$. In this way, the relevant parameter would be the density on $\Gamma$ of the *total* number of strategies $sN$. In Fig. 4 we plot $\sigma^2/N$ as a function of $z'$, at different values of $D$, $N$, and $s$, for the model without memory. We see that the correct scaling parameter is $z$ and not $z'$, since the data with different values of $s$ collapse on different curves. The same result is obtained if we perform the simulation with the memory (see [5]). The two models give once again the same results. Note from Fig. 4 that the scaling is not perfect at very low values of $z'$; that is, for very small $D$. This is just a trace of the integer nature of the model.

From what is shown above it is reasonable to conclude that in order to obtain all the crucial features of the minority game, the presence of an individual memory of the agents is irrelevant. The parameter $m$ still plays a major role, but only for being related to the dimension $D=2^m$ of the strategies space $\Gamma$. A consequence of this fact is that any attempt to explain the properties of this model, relying on the role of $m$ as a memory, can hardly be correct. On the other hand, as already said, the geometrical role of $m$ remains. Indeed, some recent attempts to give an analytic description of the model (see [4,6,9]) are only grounded in geometrical considerations about the distribution of strategies in the space $\Gamma$ and go, therefore, in our opinion, in the correct direction.

The most important result of the present Rapid Communication is the existence of a regime where the whole population of agents still behaves in a better-than-random way, even if the information they process is completely random, that is wrong if compared to the real time history. *The crucial thing is that everyone must possess the same information*. Indeed, if we invent a different past history for each different agent, no coordination emerges at all and the results are the same as if the agents were behaving randomly (this can be easily verified numerically). In other words, if each individual is processing different information, the features of the system are completely identical to the random case, irrespective of the values of $m$ and $s$.

The conclusion is the following: the crucial property is not at all the agents' memory of the real time history, but rather the fact that they all share the same information, however false or true this is. As a consequence, there is no room in this model for any kind of forecasting of the future based on the ''understanding'' of the past.

We hope this result will be useful for a future deeper understanding of this kind of adaptive system. Indeed, before trying to explain the rich structure of a quite complicated model, it is important in our opinion to clear up what the truly necessary ingredients of such a model are and what, on the contrary, is just an irrelevant complication that can be dropped. In the case of the so-called memory (or brain size, or intelligence), $m$, there also has been a problem of terminology: given the original formulation of the model, it seemed that the very nature of a variable encoding the *memory* or the *intelligence* of the agents, could warrant by itself a relevance to it [2–8], relevance which, as we have seen, was not deserved. Notwithstanding this, we still consider the present model to be very interesting and far from trivial.

Finally, let us note that the passage from a model with memory to a model without memory is equivalent to replacing a deterministic but very complicated system with a stochastic but much simpler one which, nevertheless, gives the same results as the original case and which is therefore indistinguishable from it for all practical purposes. The use of a stochastic/disordered model to mimic a deterministic/ordered one, is similar in spirit to what happens in the context of glassy systems, where some disordered models of spin glasses are often used in order to have a better understanding of structural glasses, which contain in principle no quenched disorder [10].

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